Project Suggestions

These are a list of possible projects. You need to pick one. Most require at least some programming and some will require substantial error analysis and model fitting. Show all intermediate steps in your working and ilustrate your writeup with code listings (where appropriate), data plots and tables (if necessary). Discuss all the techniques and approaches used. Remember that the project will be worth 75% of your final grade and should therefore constitute a substantial piece of work and something you are proud of. I am open to other project topics if you can present a case for doing something else. You may start work on a project immediately but at the very least you will need to tell me what you intend to do by November 18 *at the very latest*

Projects will be due one week after the last day of classes.

1. Use the cluster algorithm to study the two dimensional Ising model. To use the existing codes you will need to set the initial values of the field $\phi(x)$ to ± 1.0 and *only* call the `ClusterUpdate()` function. Extract an estimate for the critical coupling κ_c and compare to the analytic result. Try to determine the critical exponent ν by direct fits of the correlation length to $\kappa_c - \kappa$. Also study the ratio γ/ν from finite size scaling of the peak in the susceptibility. Push your simulations up to as large a lattice as you can while making sure you have the statistical errors under control.
   Hint: To eliminate some of the ambiguity in determining the average magnetization you should measure not $\sum_x \phi(x)$ but its absolute value $|\sum_x \phi(x)|$.

2. Write a Monte Carlo program for simulating the $Z(2)$ gauge model in four dimensions. You may recycle as much of the existing simulation code as you find useful. A simple Metropolis algorithm should suffice. You should measure the energy, specific heat and a set of Wilson loops. By examining the area dependence of the latter at various values of the coupling constant determine in which of the phases the model is confining. From your data make an assessment of the order of any phase transitions you observe.

3. Apply Fourier acceleration techniques to improve the efficiency of the Hybrid Monte Carlo program used for simulating the 2d scalar field model. You will need to come up with a routine for doing fast Fourier transforms FFTs. Consult Numerical Recipes for a code for this (you can use the code as a "black box" without justification). Compare your code against a non-accelerated code close to the phase transition. Estimate the dynamic critical exponent in both cases by computing the autocorrelation time as a function of lattice size for the critical system.

4. Write a program that utilizes the conjugate gradient algorithm to solve the linear system
$$A_{xx'} f_{x'} = b_x$$
where the vectors $f_x \equiv f(x)$, $b_x \equiv b(x)$ take values on a *one-dimensional* lattice and the matrix A can be defined by its action on a 1d lattice field
$$\sum_{x'} A(x,x')p(x') = p(x+1) - p(x) + \phi(x)p(x)$$

where $\phi(x)$ is a 1d scalar field. To render the algorithm stable you should premultiply both sides of the equation by A^T before using the CG algorithm. In the case where $\phi(x)$ is just a Gaussian random variable at each lattice site show examples of how the norm of the residual vector decreases as the number of iterations is increased. Check that your final solution vector is indeed a good solution of the original linear system. Consider the Hamiltonian

$$H = \sum_x \frac{1}{2}\sigma(x)(A^T A)^{-1}\sigma(x)$$

Calculate $\frac{\partial H}{\partial \phi}$ and $\frac{\partial H}{\partial \sigma}$. Hence write down the set of leapfrog equations you would need to implement a HMC simulation of this system

Optional: write a code to simulate this Hamiltonian using a HMC algorithm and the CG function you wrote in the first part of this problem.