Our Corner of the Universe
AST101, Fall 2007
KEPLER'S THIRD LAW AND THE DETERMINATION OF JUPITER'S MASS
Week of November 13

PRE-LAB ACTIVITY

In this activity you will calculate the mass of the planet Jupiter by studying the motions of the Galilean moons.

- For this activity you will need a calculator.
- Read more precisely Astronomy p. 39

Kepler's Third Law

Kepler had shown that the square of the period of a planet's orbit about the sun is proportional to the cube of the semi-major axis, or

$$1) \qquad P^2 = Ka^3$$

Where K is a constant of proportionality, P is the period of the planet expressed in years and a is the semi-major axis. Newton later showed that all of Kepler's empirical laws could be derived from his own, more fundamental, laws of motion as set forth in the *Principia*. Newton also showed that the constant of proportionality in the above formula depends upon the masses of the bodies involved:

$$2) \qquad P^2 = a^3 / M$$

Where P is in years, a is in AUs and the masses are measured in units of solar masses. The mass of the Sun (1 solar mass) is 1.99×10^{30} kg. In the above form, we can use Kepler's Third Law to determine the mass. In particular, the mass of a planet can be determined from the orbital motion of its moons. Solve Eq. 2 for M.

LAB

The Mass of Jupiter

In order to apply Newton's revision of Kepler's Third Law, we must know the semi-major axis (in AU) as well as the period of one of Jupiter's moons. We will determine each of these from the chart provided on the last page of this activity. We must first make sure we understand what information is contained in the chart. If we observe Jupiter through the telescope at midnight, on July 10, what will we see? Draw and label a picture of the telescopic view. Draw (and label) the same situation as viewed from above Jupiter's North Pole. Be sure you have the correct relative distance of the moons from Jupiter. Do the same for July 18. Check your answer with the T.A.

1) First, you obtain the semi-major axes. All measurements should be made to the nearest tenth of a millimeter and each measurement recorded. Measure the distance between the extreme points east and west of Jupiter, in the motion of Callisto and Ganymede. Divide by 2 to obtain the semi-major axes with respect to the scale of the chart.

2) To convert your measurement of the semi-major axis to A.U., you will use that Jupiter's diameter at the equator is 9.5×10^{-4} AU. Find the appropriate scale factor (# of A.U. per mm) by measuring the width of Jupiter on the chart and dividing by Jupiter's diameter given above.

3) Multiply the semi-major axis as measured in the scale of the chart by this scale factor to obtain the semi-major axis in AU.

4) Next we will find the period of Callisto and Ganymede orbits about Jupiter. Again, referring to the chart, measure the distance between the maxima of the greatest eastern or western elongation from Jupiter. Convert this number from days to years, to obtain the period P.

5) Use equation (2) to find the total mass of Jupiter.

6) Make a table containing the semi-major axes, the periods and Jupiter's mass as determined from following the motion of Callisto and Ganymede.

YOUR LAB REPORT

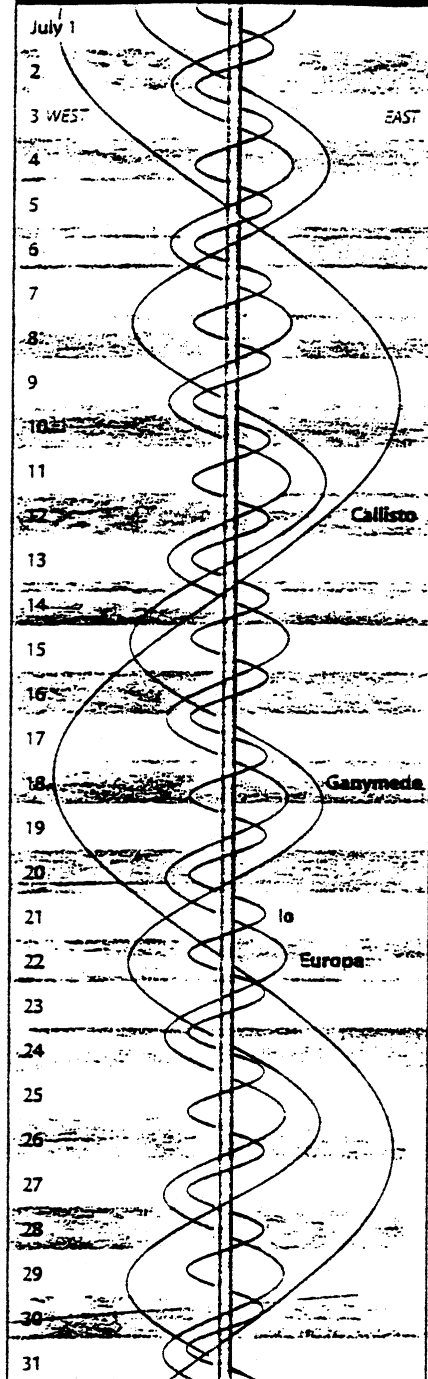
a) Discuss Kepler's Third Law and describe, fully, the procedure followed in this lab. (Use your own words!) Discuss the sources of error in your measurements.

b) What is the average of your measurements for Jupiter's mass?

c) How much more massive is Jupiter than the Earth, according to your measurements?

d) Attach the 4 diagrams of Jupiter and its moons as required in the text, as well as the table containing the results of your measurements. Use a scale with 1mm = 50,000 km

Jupiter's Satellites



The curving lines represent Jupiter's four bright satellites. Jupiter itself is the center vertical bar. Each horizontal band represents a full day, from 0^h (upper edge of band) to 24^h Universal Time. The date is given at left, and 1 mm vertically is very nearly four hours. West is left and east is right to match the view from the Northern Hemisphere in an astronomical (inverting) telescope; binocular observers can just turn the page upside down. The bottom diagram shows where the satellites disappear (d) or reappear (r) during their eclipses by Jupiter's shadow nearest midmonth.